Extending Precoloring of Fractional Coloring

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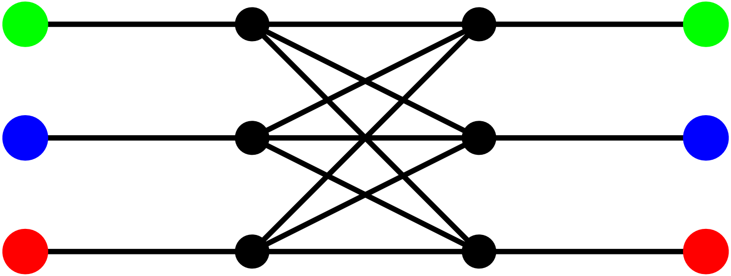
Midsummer Combinatorial Workshop XVI

Theorem (Albertson)

Let P be an independent set on an arbitrary k-colorable graph G and $d(P) \geq 4$, then every coloring of P from a set of $k+1$ colors extends to a proper $(k+1)$-coloring of G.



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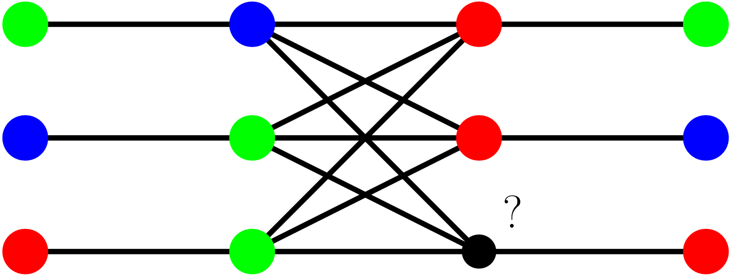

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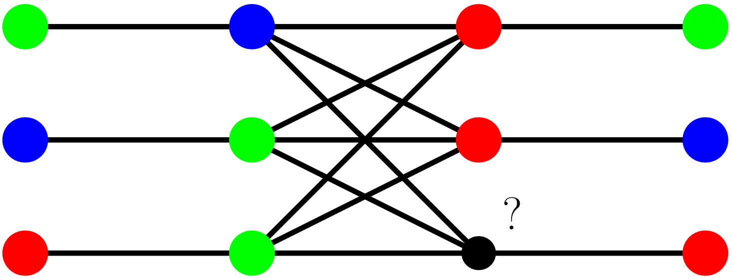

What about other types colorings?

Theorem (Circular coloring – Albertson & West JCTB'06)

Given positive integers k, d, k', d' with $k'/d' > k/d \geq 2$, let $l = \lceil kk'/(2(k'd - kd')) \rceil$. If $\chi_c(G) \leq k/d$, and $P \in V(G)$ is an independent set such that $d(P) \geq 2l$, then every precoloring of P from $Z_{k'}$ extends to a k'/d'-coloring of G.

Definition (Fractional coloring)

Let I be an interval of length d. A d-coloring of graph G is a map $f : V(G) \to \mathcal{P}(I)$ such that

$$\forall x \in V(G)\ ||f(x)|| \geq 1$$

and

$$\forall x, y \in E(G)\ f(x) \cap f(y) = \emptyset.$$

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Can I bound normal coloring by fractional coloring?

Theorem (Fractional coloring theorem)

Let P be an independent set on an arbitrary χ-colorable graph G and denote $d = min\{d(u,v) | u \neq v \in P\}$. Then every coloring of P from a set of $\chi + \varepsilon$ colors extends to a proper $(\chi + \varepsilon)$-coloring of G, where ε is selected as follows:

$$\begin{array}{lccc}
d = 0 \bmod 4: & \frac{1}{\chi+\varepsilon} & \geq & 1 - \frac{d}{4}\varepsilon \\
d = 1 \bmod 4: & \frac{1}{\chi} & \geq & 1 - \frac{d-1}{4}\varepsilon \\
d = 2 \bmod 4: & \frac{\chi-1}{\chi+\varepsilon} & \leq & 1 - \frac{d-2}{4}\varepsilon \\
d = 3 \bmod 4: & \frac{(1-\varepsilon)(1-\chi)}{\chi} & \leq & \frac{d-3}{4}\varepsilon
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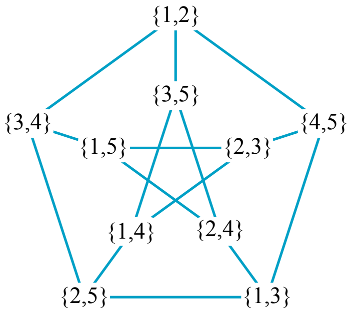
$$\chi \in \{2\} \cup [3, \infty)$$

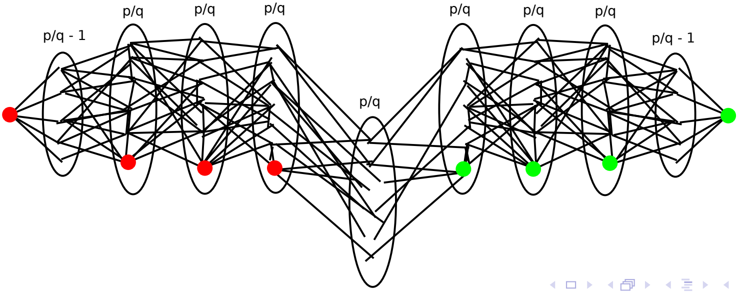
Definition ((p, q)-clique graph – Kneser graph)

Maximal graph G colored by p/q-colors using just intervals of size $1/q$ will be called (p, q)-clique



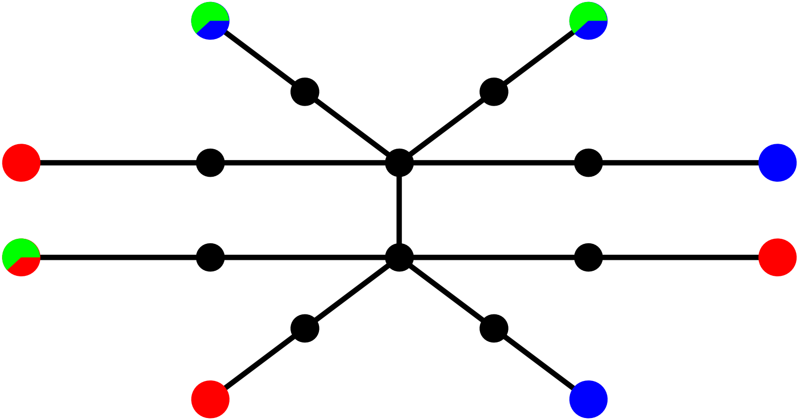
Definition (d-universal graph)

Graph H is composed of chains from (p, q)-cliques. Each vertex is connected to all vertices of next clique except its siblink. These chains have length $d - 1$ and one end of each is connected to another (p, q)-clique, the other end has one selected vertex(called 'precolored' vertex) which is disconnected from folowing clique. Any set of precolored vertices can be homomorphicaly mapped in such H.



Our theorems for $d = 4$ and $\chi = 2$ says:

$$\frac{1}{2+\varepsilon} \geq 1 - \varepsilon \Rightarrow \varepsilon = \frac{\sqrt{5}-1}{2}$$



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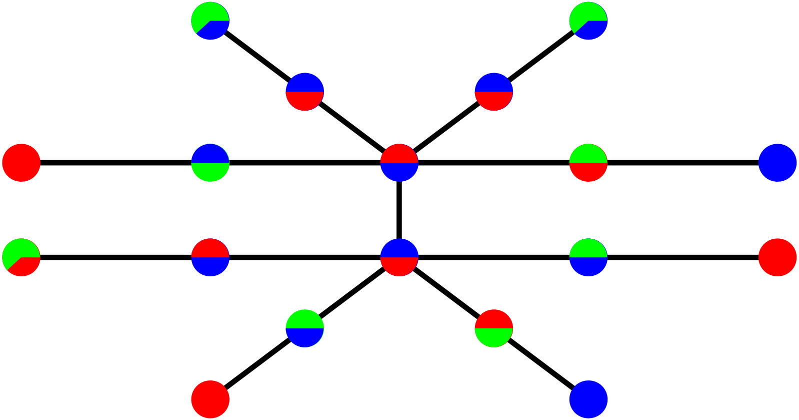

Problem ($2 < \chi < 3$)

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Questions ?